IFP-intuitionistic fuzzy soft set theory and its applications

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Abstract

In this work, we present definition of intuitionistic fuzzy parameterized (IFP) intuitionistic fuzzy soft set and its operations. Then we define IFP-aggregation operator to form IFP-intuitionistic fuzzy soft-decision-making method which allows constructing more efficient decision processes.

Keywords: Soft set, fuzzy set, intuitionistic fuzzy set, intuitionistic fuzzy soft set, intuitionistic fuzzy parameterized intuitionistic fuzz soft set, aggregation operator.

1. Introduction

Problems in economy, engineering, environmental science and social science and many fields involve data that contain uncertainties. This problems may not be successfully modeled by existing methods in classical mathematics because of various types of uncertainties. There are some well known mathematical theories for dealing with uncertainties such as; fuzzy set theory [17], soft set theory [15], intuitionistic fuzzy set theory [1], fuzzy soft set theory [12] and so on.

In 1999, Molodtsov [15] firstly introduced the soft set theory as a general mathematical tool for dealing with uncertainty and vagueness. Since then some authors studied on the operations of soft sets [2, 3, 14].

Many interesting results of soft set theory have been studied by embedding the ideas of fuzzy sets. Fuzzy Soft sets [5, 4, 9, 12, 16], intuitionistic

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fuzzy soft sets [10, 11, 13]. Firstly, fuzzy parameterized soft set and fuzzy parameterized fuzzy soft set and their operations are introduced Çağman et al. in [4, 5]. Intuitionistic fuzzy parameterized soft set is define by Çağman and Deli [7] and intuitionistic fuzzy parameterized fuzzy soft set and its operations are introduced by Çağman and Karaaslan [8].

In this paper, firstly we present preliminaries and then we introduce intuitionistic fuzzy parameterized intuitionistic fuzzy soft set and their properties. We also define IFP-aggregation operator to form $IFP-$intuitionistc fuzzy soft decision making method that allows constructing more efficient decision processes. We finally present examples which shows that the methods can be successfully applied to many problems that contain uncertainties.

2. Preliminary

In this section, we present definitions and some results of soft set, fuzzy set, fuzzy soft set, intuitionistic fuzzy set and intuitionistic fuzzy soft set theory that can be found details [1, 3, 12, 5, 11, 14, 15, 17].

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U.

Definition 1. *[3] Let U be an initial universe, $P(U)$ be the power set of U, E is the set of all parameter and $A \subseteq E$. Then, a soft set F_A on the universe U is defined by a function f_A representing a mapping*

$$f_A : E \to P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \notin A$$

Here f_A is called approximate function of soft set F_A, and the value $f_A(x)$ is a set called x-element of the soft set for all $x \in E$. It is worth nothing that the set $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set F_A over U can be represented by the set of ordered pairs

$$F_A = \{(x, f_A) : x \in E, f_A(x) \in P(U)\}$$

Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 2. *[17] Let U be a universe. A fuzzy set X over U is a set defined by a function μ_X representing a mapping*

$$\mu_X : U \to [0, 1]$$

Here, μ_X called membership function of X and the value $\mu_X(u)$ is called the grade of membership of $u \in U$. The value represents the degree of u belonging to fuzzy set X. Thus, a fuzzy set X over U can be represented as follows,

$$X = \{(u, \mu_X(u)) : u \in U, \mu_X(u) \in [0, 1]\}$$

Note that the set of all the fuzzy sets over U will be denoted by $F(U)$.

Definition 3. *[1] An intuitionistic fuzzy set (IFS) X in U is defined as an object of the following form*

$$X = \{(x, \mu_X(u), \nu_X(u)) : u \in U\},$$

where the functions $\mu_X : U \to [0, 1]$ and $\nu_X : U \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $u \in U$, respectively, and for every $u \in U$,

$$0 \leq \mu_X(u) + \nu_X(u) \leq 1.$$

In addition for all $u \in U$, $U = \{(u, 1, 0) : u \in U\}$, $\emptyset = \{(u, 0, 1) : u \in U\}$ are intuitionistic fuzzy universal and intuitionistic fuzzy empty set, respectively.

Theorem 1. *[1] Let X and Y be two intuitionistic fuzzy sets. Then,*

i. $X \subseteq Y \Leftrightarrow \forall u \in U, \mu_X(u) \leq \mu_Y(u), \nu_X(u) \geq \nu_Y(u)$
ii. $X \cap Y = \{(u, min\{\mu_X(u), \mu_Y(u)\}, max\{\nu_X(u), \nu_Y(u)\}) : u \in U\}$
iii. $X \cup Y = \{(u, max\{\mu_X(u), \mu_Y(u)\}, min\{\nu_X(u), \nu_Y(u)\}) : u \in U\}$.
iv. $X^c = \{(u, \nu_X(u), \mu_X(u)) : u \in U\}$.

Note that the set of all the fuzzy sets over U will be denoted by $\mathcal{IF}(U)$.

Definition 4. *[6] Let U be an initial universe, $\mathcal{IF}(U)$ be the set of all intuitionistic fuzzy sets over U, E be a set of all parameters and $A \subseteq E$. Then, an intuitionistic fuzzy soft set (IFS-set) γ_A over U is a function from E into $\mathcal{IF}(U)$.*

Where, the value $\gamma_A(x)$ is an intuitionistic fuzzy set over U. That is, $\gamma_A(x) = \{(u, \overline{\gamma}_{A(x)}(u), \underline{\gamma}_{A(x)}(u)) : x \in E, u \in U)\}$, where $\overline{\gamma}_{A(x)}(u)$ and $\underline{\gamma}_{A(x)}(u)$ are the membership and non-membership degrees of u to the parameter x, respectively.

Note that, the set of all intuitionistic fuzzy soft sets over U is denoted by $\mathcal{IFS}(U)$.

Definition 5. *[6] Let $A, B \subseteq E$, γ_A and γ_B be two IFS-sets. Then, γ_A is said to be an intuitionistic fuzzy soft subset of γ_B if*
(1) $A \subseteq B$ and
(2) $\gamma_A(x)$ is an intuitionistic fuzzy subset of $\gamma_B(x)$ $\forall x \in A$.

This relationship is denoted by $\gamma_A \tilde{\subseteq} \gamma_B$. Similarly, γ_A is said to be an intuitionistic fuzzy soft superset of γ_B, if γ_B is an intuitionistic fuzzy soft subset of γ_A and denoted by $\gamma_A \tilde{\supseteq} \gamma_B$.

Definition 6. *[6] Let γ_A and γ_B be two intuitionistic fuzzy soft sets over U. Then, γ_A and γ_B are said to be intuitionistic fuzzy soft equal if and only if γ_A is an intuitionistic fuzzy soft subset of γ_B and γ_B is an intuitionistic fuzzy soft subset of γ_A, and written by $\gamma_A \tilde{=} \gamma_B$.*

Definition 7. *[6] Let γ_A be an IFS-set over $\mathcal{IF}(U)$. If $\gamma_A(x) = \emptyset$ for all $x \in E$, then γ_A is called empty IFS-set and denoted by γ_ϕ.*

Definition 8. *[6] Let γ_A be an IFS-set over $\mathcal{IF}(U)$. If $\gamma_A(x) = \{(u, 1, 0) : \forall u \in U\}$ for all $x \in A$, then γ_A is called A-universal IFS-set and denoted by $\gamma_{\hat{A}}$.*

If A=E, then the A-universal IFS-set is called universal IFS-set and denoted by $\gamma_{\hat{E}}$.

Definition 9. *[6] Let γ_A and γ_B be two IFS-sets over $\mathcal{IF}(U)$. Union of γ_A and γ_B, denoted by $\gamma_A \tilde{\cup} \gamma_B$, and is defined by*

$$\gamma_A \tilde{\cup} \gamma_B = \{(x, \gamma_{A \tilde{\cup} B}(x)) : x \in E\}$$

where

$$\gamma_{A \tilde{\cup} B}(x) = \{(u, max\{\overline{\gamma}_{A(x)}(u), \overline{\gamma}_{B(x)}(u)\}, min\{\underline{\gamma}_{A(x)}(u), \underline{\gamma}_{B(x)}(u)\}) : u \in U\}.$$

Definition 10. *[6] Let γ_A and γ_B be two IFS-set over $\mathcal{IF}(U)$. Intersection of γ_A and γ_B, denoted by $\gamma_A \tilde{\cap} \gamma_B$, and is defined by*

$$\gamma_A \tilde{\cap} \gamma_B = \{(x, \gamma_{A \tilde{\cap} B}(x)) : x \in E\}$$

where

$$\gamma_{A \tilde{\cap} B}(x) = \{(u, min\{\overline{\gamma}_{A(x)}(u), \overline{\gamma}_{B(x)}(u)\}, max\{\underline{\gamma}_{A(x)}(u), \underline{\gamma}_{B(x)}(u)\}) : u \in U\}.$$

Definition 11. *[6] Let γ_A be an IFS-set over $\mathcal{IF}(U)$. Complement of γ_A, denoted by γ_A^c, and is defined by*

$$\gamma_A^c = \{(x, \gamma_{A^c}(x)) : x \in E\}$$

where $\gamma_{A^c}(x) = \gamma_A^c(x)$ is the complement of intuitionistic fuzzy set $\gamma_A(x)$, defined by

$$\gamma_A^c(x) = \{(u, \underline{\gamma}_{A(x)}(u), \overline{\gamma}_{A(x)}(u)) : u \in U\}$$

for all $x \in E$

Definition 12. *[6]Let γ_A and γ_B be two IFS-set over $\mathcal{IF}(U)$. $\wedge-$product of γ_A and γ_B, denoted by $\gamma_A \wedge \gamma_B$, and is defined by*

$$\gamma_A \wedge \gamma_B = \{((x, y), \gamma_{A\wedge B}(x, y)) : (x, y) \in E \times E\}$$

where

$$\gamma_{A\wedge B}(x, y)) = \{(u, min(\mu_{\gamma_{A(x)}}(u), \mu_{\gamma_{B(x)}}(u))), max(\nu_{\gamma_{A(x)}}(u), \nu_{\gamma_{B(x)}}(u))) : u \in U\}$$

for all $x, y \in E$

Definition 13. *[6]Let γ_A and γ_B be two IFS-set over $\mathcal{IF}(U)$. $\vee-$product of γ_A and γ_B, denoted by $\gamma_A \vee \gamma_B$, and is defined by*

$$\gamma_A \vee \gamma_B = \{((x, y), \gamma_{A\vee B}(x, y)) : (x, y) \in E \times E\}$$

where

$$\gamma_{A\vee B}(x, y)) = \{(u, max(\mu_{\gamma_{A(x)}}(u), \mu_{\gamma_{B(x)}}(u))), min(\nu_{\gamma_{A(x)}}(u), \nu_{\gamma_{B(x)}}(u))) : u \in U\}$$

for all $x, y \in E$

Definition 14. *[5] Let U be an initial universe, E be the set of all parameters and X be a fuzzy set over E with the membership function $\mu_X : E \to [0, 1]$ and $\gamma_X(x)$ be a fuzzy set over U for all $x \in E$. Then, and $fpfs-$set Γ_X over U is a set defined by a function $\gamma_X(x)$ representing a mapping*

$$\gamma_X : E \to F(U) \text{ such that } \gamma_X(x) = \emptyset \text{ if } \mu_X(x) = 0$$

Here, γ_X is called fuzzy approximate function of ($fpfs$-set) Γ_X, and the value $\gamma_X(x)$ is a fuzzy set called $x-$element of the $fpfs-$set for all $x \in E$. Thus, an $fpfs-$set Γ_X over U can be represented by the set of ordered pairs

$$\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(U), \mu_X(x) \in [0, 1]\},$$

It must be noted that the sets of all $fpfs$-sets over U will be denoted by $FPFS(U)$.

Definition 15. *[7] Let U be an initial universe, $P(U)$ be the power set of U, E is the set of all parameters and X be a intuitionistic fuzzy set over E with the membership function $\mu_X : E \to [0,1]$ and non-membership function $\nu_X : E \to [0,1]$. Then, an $ifps-set$ F_X over U is a set defined by a function f_X representing a mapping*

$$f_X : E \to P(U) \text{ such that } f_X(x) = \emptyset \text{ if } \mu_x = 0, \nu_x = 1$$

Here, f_X is called approximate function of the $ifps$-set F_X, and the value $f_X(x)$ is a set called $x-$element of $ifps$-set for all $x \in E$. Thus, $ifps-set$ F_X over U can be represented by the set of ordered pairs

$$F_X = \{((\mu_X(x), \nu_X(x))/x, f_X(x)) : x \in E, f_X(x) \in P(U), \mu_X(x), \nu_X(x) \in [0,1]\}$$

Definition 16. *[8] Let U be an initial universe, $P(U)$ be the power set of U, E is the set of all parameters and X be a intuitionistic fuzzy set over E with the membership function $\mu_X : E \to [0,1]$ and non-membership function $\nu_X : E \to [0,1]$. Then, an $ifps-set$ F_X over U is a set defined by a function f_X representing a mapping*

$$f_X : E \to P(U) \text{ such that } f_X(x) = \emptyset \text{ if } \mu_x = 0, \nu_x = 1$$

Here, f_X is called approximate function of the $ifps$-set F_X, and the value $f_X(x)$ is a set called $x-$element of $ifps$-set for all $x \in E$. Thus, $ifps-set$ F_X over U can be represented by the set of ordered pairs

$$F_X = \{((\mu_X(x), \nu_X(x))/x, f_X(x)) : x \in E, f_X(x) \in P(U), \mu_X(x), \nu_X(x) \in [0,1]\}$$

3. *IFP*−intuitionistic fuzzy soft sets

In this section, we define intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets and their operations with examples.

Throughout this work, we use $\Omega_X, \Omega_Y, \Omega_Z, ...$,etc. for Ω-sets and $\omega_X, \omega_Y, \omega_Z, ...$,etc. for their intuitionistic fuzzy approximate function, respectively.

Definition 17. *Let U be an initial universe, E be the set of all parameters and X be an intuitionistic fuzzy set over E with the membership function $\mu_X : E \to [0,1]$ and non-membership function $\nu_X : E \to [0,1]$ and ω_X is an intuitionistic fuzzy set over U for all $x \in E$. Then, an Ω-set Ω_X over $\mathcal{IF}(U)$ is a set defined by a function $\omega_X(x)$ representing a mapping*

$$\omega_X : E \to \mathcal{IF}(U) \text{ such that } \omega_X(x) = \emptyset \text{ if } x \notin X$$

Here, ω_X is called intuitionistic fuzzy approximation of Ω-set Ω_X. $\omega_X(x)$ is an intuitionistic fuzzy set called x-element of the Ω-set for all $x \in E$. Thus, an Ω-set Ω_X over U can be represented by the set of ordered pairs

$$\Omega_X = \{((\mu_X(x), \nu_X(x))/x, \omega_X(x)) : x \in E, u \in U, \omega_X(x) \in \mathcal{IF}(U)\}$$

Note that, If $\mu_X(x) = 0, \nu_X(x) = 1$ and $\omega_X(x) = \emptyset$, we don't display such elements in the set. Also, it must be noted that the sets of all Ω-sets over $\mathcal{IF}(U)$ will be denoted by $\Omega(U)$.

Example 1. *Assume that $U = \{u_1, u_2\, u_3, u_4, u_5\}$ is an universal set and $E = \{x_1, x_2, x_3, x_4, x_5\}$ a set of parameters. If $X = \{(0.5, 0.2)/x_1, (0.6, 0.3)/x_3, (1.0, 0.0)/x_4\}$*
$\omega_X(x_1) = \{(0.7, 0.2)/u_1, (0.5, 0.4)/u_4\}$,
$\omega_X(x_2) = \emptyset$,
$\omega_X(x_3) = \{(0.4, 0.3)/u_2, (0.8, 0.1)/u_3, (0.6, 0.3)/u_5\}$
$\omega_X(x_4) = U$,
then the Ω_X is written as follow

$$\begin{aligned} \Omega_X = \{ & ((0.5, 0.2)/x_1, \{(0.7, 0.2)/u_1, (0.5, 0.4)/u_4\}), \\ & ((0.6, 0.3)/x_3, \{(0.4, 0.3)/u_2, (0.8, 0.1)/u_3, (0.6, 0.3)/u_5\}), \\ & ((1.0, 0.0)/x_4, U)\} \end{aligned}$$

Definition 18. *Let $\Omega_X \in \Omega(U)$. If $\omega_X(x) = \emptyset$ for all $x \in E$, then Ω_X is called an X-empty Ω-set, denoted by $\Omega_{\emptyset_X}$.*
If $X = \emptyset$, then the $X-$empty Ω-set ($\Omega_{\emptyset_X}$) is called empty Ω-set, denoted by $\Omega_\emptyset$. Here, $\emptyset$ mean that intuitionistic fuzzy empty set.

Definition 19. *Let $\Omega_X \in \Omega(U)$. If $\mu_X(x) = 1$, $\nu_X(x) = 0$ and $\omega_X(x) = U$ for all $x \in X$, then Ω_X is called X-universal Ω-set, denoted by $\Omega_{\tilde{X}}$.*

If X is equal to intuitionistic fuzzy universal set over E, then the X-universal Ω-set is called universal Ω-set, denoted by $\Omega_{\tilde{E}}$. Here, U mean that intuitionistic fuzzy universal set.

Example 2. *Let* $U = \{u_1, u_2, u_3, u_4\}$ *be a universal set and* $E = \{x_1, x_2, x_3, x_4\}$ *be a set of parameters. If*

$$\begin{aligned} X &= \{(0.2, 0, 5)/x_2, (0.5, 0, 3)/x_3, (1.0, 0)/x_4\} \text{ and} \\ \omega_X(x_1) &= \emptyset, \\ \omega_X(x_2) &= \{(0.5, 0.4)/u_1, (0.7, 0.3)/u_5\}, \\ \omega_X(x_3) &= \emptyset, \\ \omega_X(x_4) &= U, \end{aligned}$$

then the Ω*-set* Ω_X *is written by*

$\Omega_X = \{((0.2, 0, 5)/x_2, \{(0.5, 0.4)/u_1, (0, 1.0)/u_2, (0, 1.0)/u_3, (0.7, 0.3)/u_4\}), ((0, 5.0, 3)/x_3, \emptyset), ((1.0, 0)/x_4, U)\}$.

If $Y = \{(1.0, 0)/x_1, (0.7, 0.2)/x_4\}$ *and* $\omega_Y(x_1) = \emptyset$, $\omega_Y(x_4) = \emptyset$ *then the* Ω*-set* Ω_Y *is an* Y*-empty* Ω*-set, i.e.,* $\Omega_Y = \Omega_{\Phi_Y}$.

If $Z = \{(1.0, 0)/x_1, (1.0, 0)/x_2\}$, $\omega_Z(x_1) = U$, *and* $\omega_Z(x_2) = U$, *then the* Ω*-set* Ω_Z *is* Z*-universal* Ω*-set, i.e.,* $\Omega_Z = \Omega_{\tilde{Z}}$.

If $X = E$ *and* $\omega_X(x_i) = U$ *for all* $x_i \in E$, *where* $i = 1, 2, 3, 4$, *then the* Ω*-set* Ω_X *is a universal* Ω*-set, i.e.,* $\Omega_X = \Omega_{\tilde{E}}$.

Definition 20. *Let* $\Omega_X, \Omega_Y \in \Omega(U)$. *Then,* Ω_X *is an* Ω*-subset of* Ω_Y, *denoted by* $\Omega_X \tilde{\subseteq} \Omega_Y$, *if* $\mu_X(x) \leq \mu_Y(x), \nu_X(x) \geq \nu_X(x)$ *and* $\omega_X(x) \subseteq \omega_Y(x)$ *for all* $x \in E$.

Proposition 1. *Let* $\Omega_X, \Omega_Y \in \Omega(U)$. *Then,*

(i) $\Omega_X \tilde{\subseteq} \Omega_{\tilde{E}}$

(ii) $\Omega_{\Phi_X} \tilde{\subseteq} \Omega_X$

(iii) $\Omega_\Phi \tilde{\subseteq} \Omega_X$

(iv) $\Omega_X \tilde{\subseteq} \Omega_X$

(v) $\Omega_X \tilde{\subseteq} \Omega_Y$ *and* $\Omega_Y \tilde{\subseteq} \Omega_Z \Rightarrow \Omega_X \tilde{\subseteq} \Omega_Z$

Proof. They can be proved easily by using the fuzzy approximate and membership functions of the Ω-sets.

Definition 21. *Let $\Omega_X, \Omega_Y \in \Omega(U)$. Then, Ω_X and Ω_Y are $\Omega-$equal, written as $\Omega_X = \Omega_Y$, if and only if $\mu_X(x) = \mu_Y(x)$, $\nu_X(x) = \nu_Y(x)$ and $\omega_X(x) = \omega_Y(x)$ for all $x \in E$.*

Proposition 2. *Let $\Omega_X, \Omega_Y, \Omega_Z \in \Omega(U)$. Then,*

(i) $\Omega_X = \Omega_Y$ *and* $\Omega_Y = \Omega_Z \Leftrightarrow \Omega_X = \Omega_Z$

(ii) $\Omega_X \widetilde{\subseteq} \Omega_Y$ *and* $\Omega_Y \widetilde{\subseteq} \Omega_X \Leftrightarrow \Omega_X = \Omega_Y$

Definition 22. *Let $\Omega_X \in \Omega(U)$. Then the complement of Ω_X, denoted by $\Omega_X^{\tilde{c}}$, is defined by*

$$\Omega_X^{\tilde{c}} = \{((\nu_X(x), \mu_X(x))/x, \omega_X^c(x)) : x \in E, \omega_X^c(x) \in \mathcal{IF}(U)\}$$

where $\omega_X^c(x)$ is complement of the intuitionistic fuzzy set $\omega_X(x)$, that is, $\omega_X^c(x) = \omega_{X^c}(x)$ for every $x \in E$.

Proposition 3. *Let $\Omega_X \in \Omega(U)$. Then,*

(i) $(\Omega_X^{\tilde{c}})^{\tilde{c}} = \Omega_X$

(ii) $\Omega_\Phi^{\tilde{c}} = \Omega_{\tilde{E}}$

Proof. By using the intuitionistic fuzzy approximate, membership functions and nonmembership functions of the Ω-sets, the proof is straightforward.

Definition 23. *Let $\Omega_X, \Omega_Y \in \Omega(U)$. Then, union of Ω_X and Ω_Y, denoted by $\Omega_X \widetilde{\cup} \Omega_Y$, is defined by*

$$\Omega_X \cup \Omega_Y = \{((\mu_{X\widetilde{\cup}Y}(x), \nu_{X\widetilde{\cap}Y}(x))/x, \omega_{X\widetilde{\cup}Y}(x)) : x \in E\}.$$

Here,

$$\mu_{X\widetilde{\cup}Y}(x) = \max\{\mu_X(x), \mu_Y(x)\}, \nu_{X\widetilde{\cup}Y}(x) = \min\{\nu_X(x), \nu_Y(x)\} \text{ and}$$

$$\omega_{X\widetilde{\cup}Y}(x) = \omega_X(x) \cup \omega_Y(x), \text{for all } x \in E.$$

Note that here $\omega_X(x)$ and $\omega_Y(x)$ are intuitionistic fuzzy sets. Thus, in operations of between $\omega_X(x)$ and $\omega_Y(x)$, we use the operations of intuitionistic fuzzy sets.

Proposition 4. *Let $\Omega_X, \Omega_Y, \Omega_Z \in \Omega(U)$. Then,*

(i) $\Omega_X \widetilde{\cup} \Omega_X = \Omega_X$

(ii) $\Omega_X \widetilde{\cup} \Omega_\Phi = \Omega_X$

(iii) $\Omega_X \widetilde{\cup} \Omega_{\tilde{E}} = \Omega_{\tilde{E}}$

(iv) $\Omega_X \widetilde{\cup} \Omega_Y = \Omega_Y \widetilde{\cup} \Omega_X$

(v) $(\Omega_X \widetilde{\cup} \Omega_Y) \widetilde{\cup} \Omega_Z = \Omega_X \widetilde{\cup} (\Omega_Y \widetilde{\cup} \Omega_Z)$

Proof. The proofs can be easily obtained from Definition 23.

Definition 24. *Let $\Omega_X, \Omega_Y \in \Omega(U)$. Then, intersection of Ω_X and Ω_Y, denoted by $\Omega_X \widetilde{\cap} \Omega_Y$, is defined by*

$$\Omega_X \cap \Omega_Y = \{((\mu_{X\widetilde{\cap}Y}(x), \nu_{X\widetilde{\cup}Y}(x))/x, \omega_{X\widetilde{\cap}Y}(x)) : x \in E\}.$$

Here,

$$\mu_{X\widetilde{\cap}Y}(x) = \min\{\mu_X(x), \mu_Y(x)\}, \nu_{X\widetilde{\cap}Y}(x) = \max\{\nu_X(x), \nu_Y(x)\}$$

and

$$\omega_{X\widetilde{\cap}Y}(x) = \omega_X(x) \cap \omega_Y(x) \, \textit{for all } x \in E.$$

Note that here $\omega_X(x)$ and $\omega_Y(x)$ are intuitionistic fuzzy sets. Thus, in operations of between $\omega_X(x)$ and $\omega_Y(x)$, we use the operations of intuitionistic fuzzy sets.

Proposition 5. *Let $\Omega_X, \Omega_Y, \Omega_Z \in \Omega(U)$. Then,*

(i) $\Omega_X \widetilde{\cap} \Omega_X = \Omega_X$

(ii) $\Omega_X \widetilde{\cap} \Omega_\Phi = \Omega_\Phi$

(iii) $\Omega_X \widetilde{\cap} \Omega_{\tilde{E}} = \Omega_X$

(iv) $\Omega_X \widetilde{\cap} \Omega_Y = \Omega_Y \widetilde{\cap} \Omega_X$

(v) $(\Omega_X \widetilde{\cap} \Omega_Y) \widetilde{\cap} \Omega_Z = \Omega_X \widetilde{\cap} (\Omega_Y \widetilde{\cap} \Omega_Z)$

Proof. The proofs can be easily obtained from Definition 24.

Remark 1. *Let* $\Omega_X \in \Omega(U)$*. If* $\Omega_X \neq \Omega_\Phi$ *or* $\Omega_X \neq \Omega_{\tilde{E}}$*, then* $\Omega_X \widetilde{\cup} \Omega_X^{\tilde{c}} \neq \Omega_{\tilde{E}}$ *and* $\Omega_X \widetilde{\cap} \Omega_X^{\tilde{c}} \neq \Omega_\Phi$*.*

Proposition 6. *Let* $\Omega_X, \Omega_Y \in \Omega(U)$*. Then De Morgan's laws are valid*

(i) $(\Omega_X \widetilde{\cup} \Omega_Y)^{\tilde{c}} = \Omega_X^{\tilde{c}} \widetilde{\cap} \Omega_Y^{\tilde{c}}$

(ii) $(\Omega_X \widetilde{\cap} \Omega_Y)^{\tilde{c}} = \Omega_X^{\tilde{c}} \widetilde{\cup} \Omega_Y^{\tilde{c}}$

PROOF. Firstly, for all $x \in E$,

$$\begin{aligned} i.\omega_{(X\widetilde{\cup}Y)^{\tilde{c}}}(x) &= \omega^c_{X\widetilde{\cup}Y}(x) \\ &= (\omega_X(x) \cup \omega_Y(x))^c \\ &= (\omega_X(x))^c \cap (\omega_Y(x))^c \\ &= \omega^c_X(x) \cap \omega^c_Y(x) \\ &= \omega_{X^{\tilde{c}}}(x) \cap \omega_{Y^{\tilde{c}}}(x) \\ &= \omega_{X^{\tilde{c}}\widetilde{\cap}Y^{\tilde{c}}}(x). \end{aligned}$$

and

$$\begin{aligned} \Omega_X \widetilde{\cup} \Omega_Y &= \{(\max\{\mu_X(x), \mu_Y(x)\}, \min\{\nu_X(x), \nu_Y(x))\})/x, \omega_{X\tilde{\cup}Y}(x) \\ &\quad : x \in E\} \\ (\Omega_X \widetilde{\cup} \Omega_Y)^{\tilde{c}} &= \{((\min\{\nu_X(x), \nu_Y(x)\}, \max\{\mu_X(x), \mu_Y(x)\})/x, \omega_{(X\tilde{\cup}Y)^c}(x)) : \\ &\quad x \in E\} \\ &= \{((\min\{\nu_X(x), \nu_Y(x)\}, \max\{\mu_X(x), \mu_Y(x)\})/x, \omega_{X^c\tilde{\cap}Y^c}(x)) : \\ &\quad x \in E\} \\ &= \{((\nu_X(x), \mu_X(x))/x, \omega_{X^c}(x)) : x \in E\} \\ &\cap \{((\nu_Y(x), \mu_Y(x)))/x, \omega_{Y^c}(x)) : x \in E\} \\ &= \Omega_X^{\tilde{c}} \widetilde{\cap} \Omega_Y^{\tilde{c}} \end{aligned}$$

The proof of *ii.* can be made similarly.

Proposition 7. *Let* $\Omega_X, \Omega_Y, \Omega_Z \in \Omega(U)$*. Then,*

(i) $\Omega_X \widetilde{\cup} (\Omega_Y \widetilde{\cap} \Omega_Z) = (\Omega_X \widetilde{\cup} \Omega_Y) \widetilde{\cap} (\Omega_X \widetilde{\cup} \Omega_Z)$

(ii) $\Omega_X \widetilde{\cap} (\Omega_Y \widetilde{\cup} \Omega_Z) = (\Omega_X \widetilde{\cap} \Omega_Y) \widetilde{\cup} (\Omega_X \widetilde{\cap} \Omega_Z)$

PROOF. For all $x \in E$,

$$
\begin{aligned}
i.\ \mu_{X\tilde{\cup}(Y\tilde{\cap}Z)}(x) &= \max\{\mu_X(x), \mu_{Y\tilde{\cap}Z}(x)\} \\
&= \max\{\mu_X(x), \min\{\mu_Y(x), \mu_Z(x)\}\} \\
&= \min\{\max\{\mu_X(x), \mu_Y(x)\}, \max\{\mu_X(x), \mu_Z(x)\}\} \\
&= \min\{\mu_{X\tilde{\cup}Y}(x), \mu_{X\tilde{\cup}Z}(x)\} \\
&= \mu_{(X\tilde{\cup}Y)\tilde{\cap}(X\tilde{\cup}Z)}(x)
\end{aligned}
$$

and

$$
\begin{aligned}
\nu_{X\tilde{\cup}(Y\tilde{\cap}Z)}(x) &= \min\{\nu_X(x), \nu_{Y\tilde{\cap}Z}(x)\} \\
&= \min\{\nu_X(x), \max\{\nu_Y(x), \nu_Z(x)\}\} \\
&= \min\{\max\{\nu_X(x), \nu_Y(x)\}, \max\{\nu_X(x), \nu_Z(x)\}\} \\
&= \min\{\nu_{X\tilde{\cup}Y}(x), \nu_{X\tilde{\cup}Z}(x)\} \\
&= \nu_{(X\tilde{\cup}Y)\tilde{\cap}(X\tilde{\cup}Z)}(x)
\end{aligned}
$$

$$
\begin{aligned}
\omega_{X\tilde{\cup}(Y\tilde{\cap}Z)}(x) &= \omega_X(x) \cup \omega_{Y\tilde{\cap}Z}(x) \\
&= \omega_X(x) \cup (\omega_Y(x) \cap \omega_Z(x)) \\
&= (\omega_X(x) \cup \omega_Y(x)) \cap (\omega_X(x) \cup \omega_Z(x)) \\
&= \omega_{X\tilde{\cup}Y}(x) \cap \omega_{X\tilde{\cup}Z}(x) \\
&= \omega_{(X\tilde{\cup}Y)\tilde{\cap}(X\tilde{\cup}Z)}(x)
\end{aligned}
$$

The proof of *ii.* can be made in a similar way.

4. Decision making method

The approximate function of an Ω-set is intuitionistic fuzzy set. The Ω_{agg} on the intuitionistic fuzzy sets is an operation by which several approximate functions of an Ω-set are combined to produce a single intuitionistic fuzzy set that is the aggregate intuitionistic fuzzy set of the Ω-set. Once an aggregate intuitionistic fuzzy set has been arrived at, it may be necessary to choose the best single crisp alternative from this set. Therefore, we can construct a decision-making method by the following algorithm.

Step 1. Construct an Ω-set Ω_X over U,

Step 2. Find the aggregate intuitionistic fuzzy set Ω_X^* of Ω_X,

Step 3. Find $max(u) = max\{\mu_{\Omega_X^*}(u) : u \in U\}$ and $min(v) = min\{\nu_{\Omega_X^*}(v) : v \in U\}$

Step 4. Find $\alpha \in [0,1]$ such that $(max(u), \alpha)/u \in \Omega_X^*$ and $\beta \in [0,1]$ such that $(\beta, min(v))/v \in \Omega_X^*$

Step 5. Find $\frac{max(u)}{max(u)+\alpha} = \alpha'$ and $\frac{\beta}{min(v)+\beta} = \beta'$

Step 6. Opportune element of U is denoted by $Opp(u)$ and it is chosen as follow

$$Opp(u)=\begin{cases} u, if\ \alpha' > \beta' \\ v, if\ \beta' < \alpha' \end{cases}$$

Example 3. *In this example, we present an application for the Ω-decision-making method.*

Let us assume that a company wants to fill a position. There are five candidates who form the set of alternatives, $U = \{u_1, u_2, u_3, u_4, u_5\}$. The choosing committee consider a set of parameters, $E = \{x_1, x_2, x_3, x_4\}$. For $i = 1, 2, 3, 4, 5$, the parameters x_i stand for "experience", "computer knowledge", "young age" and "good speaking", respectively.

After a serious discussion each candidate is evaluated from point of view of the goals and the constraint according to a chosen subset $X = \{(0.7, 0.2)/x_2, (0.8, 0, 2)/x_3, (0.6, 0.3)/x_4\}$ of E. Finally, the committee constructs the following Ω-set over U.

Step 1: *Let the constructed Ω-set, Ω_X, be as follows,*

$$\begin{aligned}\Omega_X = \ & \Big\{(0.7, 0.2)/x_2, \{(0.4, 0.3)/u_1, (0.7, 0.3)/u_2, (0.6, 0.2)/u_3, (0.1, 0.5)/u_4, \\ & (0.9, 0.1)/u_5\}), ((0.8, 0.2)/x_3, \{0.8, 0.1)/u_1, (0.8, 0.2)/u_2, (0.5, 0.3)/u_3, \\ & (0.7, 0.3)/u_4\}), ((0.6, 0.3)/x_4, \{(0.5, 0.5)/u_1, (0.6, 0.1)/u_3, (0.3, 0, 6)/u_5\})\Big\}\end{aligned}$$

Step 2: *The aggregate intuitionistic fuzzy set can be found as,*

$$\begin{aligned}\Omega^*_X \ = \ & \{(0.318, 0.057)/u_1, (0.248, 0.100)/u_2, (0.295, 0.033)/u_3, (0.158, 0.115)/u_4, \\ & (0.203, 0.100)/u_5\}\end{aligned}$$

Step 3:$max(u) = 0.318$ *and* $min(v) = 0.033$

Step 4: $(0.318, 0.057)/u_1 \in \Omega^*_X$ *and* $(0.295, 0.033)/u_3 \in \Omega^*_X$

Step 5: $\alpha' = \frac{0.318}{0.318+0.057} = 0.848$ *and* $\beta' = \frac{0.295}{0.295+0.033} = 0.899$

Step 6:*Since* $\alpha' < \beta'$, $Opp(u) = u_3$.

Note that, although membership degree of u_1 is bigger than u_3, opportune element of U is u_3. This example show how the effect on decision making of non-membership degrees of elements.

5. Conclusion

In this paper, firstly we have defined IFP-intuitionistic fuzzy soft sets and their operations. Then we have presented a decision making methods on the IFP-intuitionistic fuzzy soft set theory. Finally, we have provided an example that demonstrating that this method can successfully work. It can be applied to problems of many fields that contain uncertainty.

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